BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B70 07089

SUBJECT: Optimization of Launch Vehicle Angle of Attack by Steepest Descent Method - Case 310

DATE: July 28, 1970

FROM: D. G. Estberg

ABSTRACT

This memorandum presents a method of obtaining the launch vehicle angle-of-attack profile such that the payload injected onto a translunar trajectory is maximized. The Saturn V can inject about 5,000 lbs. of additional payload by using the optimum angle of attack.

The steepest-descent method was used to determine the optimum angle-of-attack profile. This method, which is derived from the calculus of variations, is an iterative numerical technique of improving an input control variable (the angle of attack) such that the payoff function (the injected payload) is optimized.

The angle of attack is currently kept zero in order to minimize the bending moment on the launch vehicle. Optimized results obtained so far are for an unconstrained angle of attack, but a steepest-descent computer program is also being developed to obtain the optimum angle-of-attack profile which is constrained such that the maximum bending moment does not exceed an established value.

BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B70 07089

SUBJECT: Optimization of Launch Vehicle Angle of Attack by Steepest-Descent Method - Case 310

DATE: July 28, 1970

FROM: D. G. Estberg

MEMORANDUM FOR FILE

I. INTRODUCTION

Currently, as the Saturn V launch vehicle ascends through the atmosphere, the angle of attack is kept zero. This memorandum presents a method of finding the angle-of-attack profile such that the payload injected onto a translunar trajectory* is maximized. The angle of attack (α) is the angle between the vehicle longitudinal axis and its velocity vector relative to the atmosphere.

The steepest-descent method, which is a numerical technique derived from calculus-of-variations theory, was used to find the optimum angle of attack. Several deviations from the standard steepest-descent method were made in order to apply it to angle-of-attack optimization.

The reason for currently keeping α zero is to minimize the bending moment on the vehicle. A measure of the bending moment imposed is given by $q|\alpha|$ where q is the dynamic pressure. Therefore, it is desired to optimize α with a constraint on $q|\alpha|$. For simplicity this constraint has not yet been used, although the steepest-descent method allows such a constraint to be imposed, and computer programming for this extension has been done.

II. SIMULATION OF THE LAUNCH VEHICLE ASCENT

In the BCMASP (Bellcomm Apollo Simulation Program) target mode simulation of the Saturn V ascent from launch to earth orbit, the angle of attack of the launch vehicle is kept zero** during first stage flight. At 12 sec after launch (just after tower clearance), the vehicle longitudinal axis is instantaneously rotated eastward. The amount of this rotation, called

---

*All results discussed in this memorandum for angle-of-attack optimization were generated to maximize payload inserted into earth orbit. These results were converted by assuming that the additional payload inserted into earth orbit is twice that injected onto a translunar trajectory.

**The procedure for doing this is called gravity turn.

the kick angle ($\Delta\theta_1$), is determined such that the injected payload is maximized. After the launch escape system is jettisoned during the second stage flight, another instantaneous rotation ($\Delta\theta_2$) takes place, and the inertial pitch rate is set equal to a constant (ω_{p2}). The values of $\Delta\theta_2$ and ω_{p2} are determined such that two terminal conditions are satisfied: orbital altitude and zero flight path angle (the angle between local horizontal and the inertial velocity vector). The other condition required for orbital insertion, orbital velocity, is automatically satisfied by cutting off the thrusting when this velocity is achieved.

A computer simulation similar to BCMASP is considered in this memorandum. A nonzero angle of attack is used from 12 sec after launch until first stage cutoff to optimize payload and to help satisfy terminal conditions. The assumptions used in the simulation are essentially the same as those used in BCMASP; however, some simplifications were made to make the implementation of angle-of-attack optimization easier. The most significant differences from BCMASP are as follows:

1. A two dimensional, inertial, rectangular coordinate system in the orbital plane was used instead of a three dimensional coordinate system. This assumes that the vehicle always remains in a plane.

2. In order to smooth the empirical tables of the lift and drag coefficients for the Saturn V vehicle, polynominal fits were used. These functions facilitated finding the derivatives of lift and drag, which are required by the steepest-descent method.

3. It was assumed that the atmosphere only extends up to the altitude of first stage cutoff (about 200,000 ft) instead of the altitude at which the launch escape system is jettisoned (about 300,000 ft).

III. THE STEEPEST-DESCENT METHOD

The physical problem of launch vehicle ascent can be described mathematically by a differential equation that depends on a control variable (angle of attack). The solution to this equation, which is called the state vector, must also satisfy boundary conditions (initial conditions plus final conditions for orbit insertion). It is desired to determine the control variable such that a payoff function (injected payload) is optimized. For this problem, the calculus-of-variations theory does not show how to find an optimum control, but only gives a condition that is useful in testing whether or not a given control is optimum. The Hamiltonian condition, which must be satisfied at each point along the trajectory if a control is optimum, is

$$H_\alpha = 0,$$

where the subscript α denotes partial differentiation with respect to the control and H is the Hamiltonian. The Hamiltonian is a function of an adjoint vector, the state vector, the control, and a constant vector ν that is the sensitivity of the payoff to the terminal-condition errors. The adjoint vector must be found by integrating the adjoint differential equations backwards starting with the transversality conditions.

The steepest-descent method can be derived from the calculus of variations (Reference 1, 2 or 3). Given a nonoptimum control, α, the steepest-descent method shows how to make an efficient change in this control, $\delta\alpha$. In the derivation only first order variations are kept so that $\delta\alpha$ only produces the desired result if all changes produced are small enough to be considered linear. If the problem were linear, steepest descent predicts that the Hamiltonian condition would be satisfied if α were changed by an amount a, which is a function of time, and that any current terminal errors would be corrected if α were changed by an amount b. In order to solve nonlinear problems a and b are multiplied by factors P and dP, respectively, where $0 < P \leq 1$ and $0 < dP \leq 1$. Therefore, the steepest-descent equation for $\delta\alpha$ is

$$\delta\alpha = a * P + b * dP. \qquad (1)$$

It is only necessary to input dP, which is commonly called the step size, since P can be determined from dP by restricting the change in control allowed by putting a maximum on the integral of $(\delta\alpha)^2$. Of course, it is desirable to set the step size dP as large as possible yet small enough to satisfy the linearity condition. The terms a and b are functions of the adjoint vector, the state vector, the control variable, and integrals of these variables; b is also a function of the current terminal errors.

IV. PROGRAMMING OF STEEPEST DESCENT FOR ANGLE-OF-ATTACK OPTIMIZATION

The procedure required to obtain a correction in the control, $\delta\alpha$, by the steepest-descent method is as follows:

(1) Initialize the state vector.

(2) Integrate the equations of motion forward in time using the present control (from an input table) and generating tables of the state vector.

(3) Determine the payoff (injected payload) and the terminal-condition errors.

(4) Determine the final conditions on the adjoint vector using the state-vector final conditions.

(5) Integrate the equations for the adjoint vector backward in time using the control and state-vector tables and generating tables of the adjoint vector. Also evaluate the integrals required in the steepest-descent equation.

(6) Using the terminal-condition errors, the control and adjoint-vector tables, and the integrals found in the previous step, generate tables of the coefficients a and b in the steepest-descent equation.

(7) Obtain the correction in the control from equation (1) using the tables of a and b from the previous step and the input step size dP.

The following paragraphs discuss the changes that were made to the standard steepest-descent method in order that it work for angle-of-attack optimization. Some of these changes affect the procedure in finding $\delta\alpha$ as described in the previous paragraph. These changes are reflected in the flow diagram of Figure 1; the original seven steps are numbered.

Standard steepest descent uses only one input step size as discussed in Section III. Due to numerical difficulties in finding the integral of $(\delta\alpha)^2$, the single-step-size method was found impractical for this physical application. Therefore, it is necessary to input both step sizes, P and dP.

It is usually quite difficult to choose fixed step sizes that will continue to work efficiently if several cycles ($\delta\alpha$'s) are required to optimize the control. Therefore, an automatic step sizing procedure is used in the steepest-descent program for α optimization. For the first determination of $\delta\alpha$, an input step size is used. Then in the second pass there is a branching after step (3) of the procedure listed above. A new step size is tried in step (7), and then steps (1) through (3) are repeated again. This cycle is repeated until an appropriate step size is found before proceeding on to step (4). Because there are two step sizes to be found, dP is found by an inner loop and P by an outer loop. The size of dP is found so as to minimize the sum of the squares of the terminal errors, and P is found so that the injected payload is maximized.

In theory it should be possible to correct terminal errors and optimize the payoff by adjusting the control variable, but, for the physical problem considered here, this was not completely possible. Only terminal errors larger than a certain minimum (because of numerical difficulties) and smaller than a certain maximum (because of the nonlinearity of the problem) can be corrected. In addition, in this acceptable region, only one of the terminal conditions (either orbital altitude or zero flight path angle) can be corrected because they cannot be made to respond independently to changes in angle of attack. Therefore, it was necessary to use the parameter controls $\Delta\theta_2$ and ω_{p2} to satisfy the terminal conditions. The values of these parameters could be found by the same method used in BCMASP, but it was found more efficient to use a steepest-descent method. An equation of the same form as equation (1) is used with $P = 0$; b is a function of the terminal errors and integrals of the same adjoint vector as used in equation (1). Since $\Delta\theta_2$ and ω_{p2} are now used to correct terminal errors, dP in equation (1) can be set equal to zero.

V. PRELIMINARY RESULTS

The terminal conditions and the payoff are much more sensitive to changes in α early in the trajectory than to changes later. Therefore, it is easier, though less effective, to correct terminal errors and to optimize payoff by implementing an angle-of-attack profile later in the trajectory while constraining α to be zero in the earlier part of the trajectory. This procedure was used while debugging the steepest-descent program with the goal of moving up the time at which optimization is started until it is at 12 sec after launch.

Figure 2 shows the additional payload that can be injected onto a translunar trajectory by using an unconstrained optimum angle of attack starting at various times after launch during first stage flight.* These results indicate that payload can be added if optimization is started before about 100 sec after launch. The payload that can be added increases more and more rapidly as optimization is started earlier. For optimization started at 12 sec after launch, about 5,000 lbs of additional payload can be injected onto a translunar trajectory. In order to obtain these results by the steepest-descent method, the following number of interation cycles ($\delta\alpha$'s) and charge units on the Univac 1108 computer were required:

Optimization Started	Cycles	Charge
100 sec	4	45
54.5 sec	4	100
22 sec	29	700

Many improvements remain to be made to increase the efficiency and reliability of the steepest-descent method as applied to launch vehicle payload optimization. At present it does not converge using automatic step sizing for optimization started at 12 sec after launch; several computer runs are necessary to obtain the optimum. Convergence could be obtained much faster if a method more sophisticated than steepest-descent could be used. Implementation of such a method, called Min-H (Reference 3), has been tried but has not yet been successful.

VI. CONCLUSION

The steepest-descent method can be used to optimize the angle-of-attack profile such that the launch vehicle payload injected onto a translunar trajectory is maximized. For unconstrained angle-of-attack, the additional payload that can be injected is about 5,000 lbs.

2013-DGE-slr

D. Esteberg

D. G. Esteberg

Attachments

---

*$\Delta\theta_1$ has not been optimized in these results, so for optimization started later than 12 sec after launch, a few hundred more pounds of payload could be injected onto a translunar trajectory.

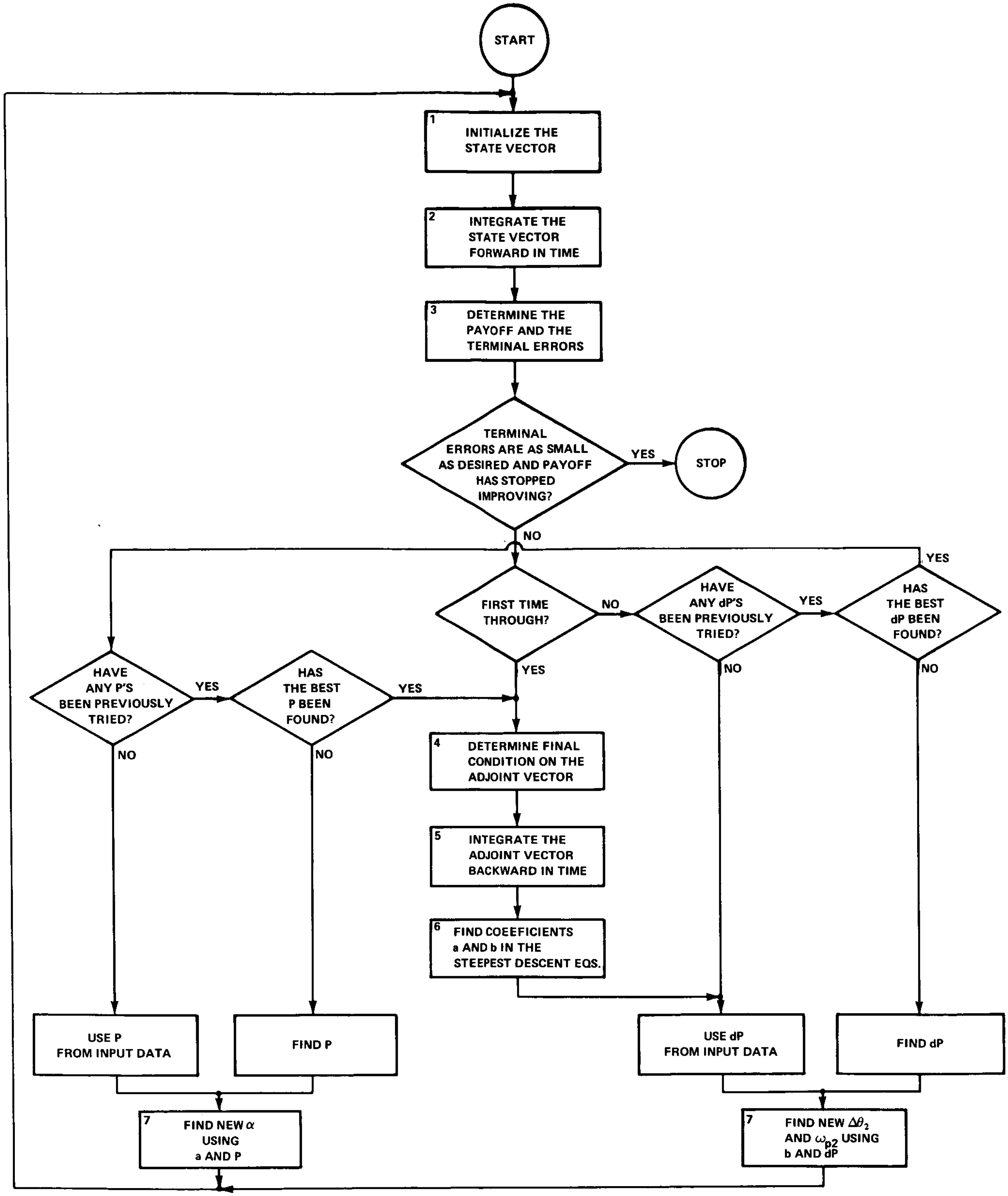

FIGURE 1 - FLOW DIAGRAM FOR STEEPEST-DESCENT METHOD WITH AUTOMATIC STEP SIZING

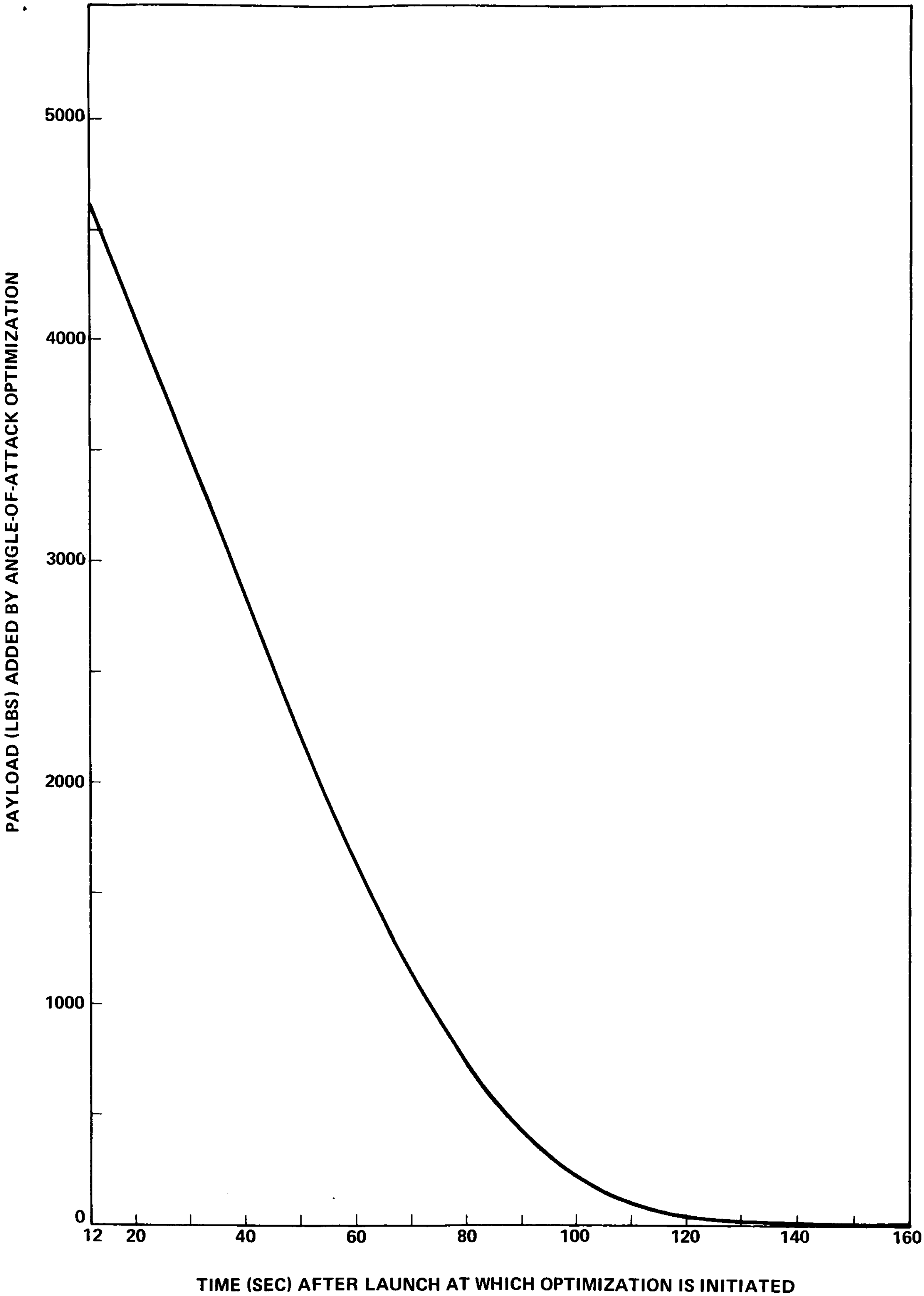

FIGURE 2 - ADDITIONAL PAYLOAD INJECTED ONTO A TRANSLUNAR TRAJECTORY BY THE SATURN V USING AN OPTIMIZED ANGLE-OF-ATTACK PROFILE INITIATED AT VARIOUS TIMES AFTER LAUNCH

REFERENCES

1. Bryson, A. E. and Denham, W. F., "A Steepest-Ascent Method for Solving Optimum Programming Problems," Journal of Applied Mechanics, Vol. 29, pp. 247-257, June, 1962.

2. Gevarter, W. B., "Optimization of Terminal Descent," Bellcomm Technical Memornadum 69-2014-6, September 3, 1969.

3. Gottlieb, Robert G., "Rapid Convergence to Optimum Solutions Using a Min-H Strategy," Preprints of the Joint Automatic Control Conference, pp. 167-176, August, 1966.

BELLCOMM, INC.

Subject: Optimization of Launch Vehicle Angle of Attack by Steepest-Descent Method - Case 310

From: D. G. Estberg

Distribution List

Bellcomm, Inc.

D. R. Anselmo
A. P. Boysen, Jr.
J. O. Cappellari, Jr.
W. G. Heffron
N. W. Hinners
D. B. James
J. L. Marshall, Jr.
K. E. Martersteck
J. Z. Menard
P. E. Reynolds
R. V. Sperry
R. L. Wagner
All Members Department 2013
Central File
Department 1024 File
Library

Abstract Only to

J. P. Downs
D. R. Hagner
I. M. Ross
J. W. Timko
M. P. Wilson